Scaling rules for the design of a narrow-band grating filter at the focus of a free-space beam

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ABSTRACT

The synthesis of a narrow band, wide angular aperture 1D grating filter exhibiting close to 100% reflection of a focused beam is developed analytically on the basis of a phenomenological coupled wave representation.

Keywords : coupled modes, diffraction gratings, photonic crystals, resonant reflection.

1. INTRODUCTION

The elementary device considered in the present paper is represented in perspective in figure 1. It consists of an infinite slab of high index material of height h pierced with periodic slits of parallel walls. The slit period is Λ, the width of the lines is fΛ where f is the fill factor, the width of the slits is (1-f)Λ. The slits and the surrounding space are filled with a low index material which can be air. The incident beam has a Gaussian profile of width w; w contains a restricted number of grating periods only, typically 10 to 20. The incidence is normal. The polarization of the electric field is parallel to the slits. The optical function to be achieved by the device is high efficiency, narrow band reflection of the incident beam and high transmission outside the reflection band. The aim of the paper is to give an intuitive way of representing the device operation so as to provide a quick and physically meaningfull design procedure.

2.THE RESONANT STRUCTURE

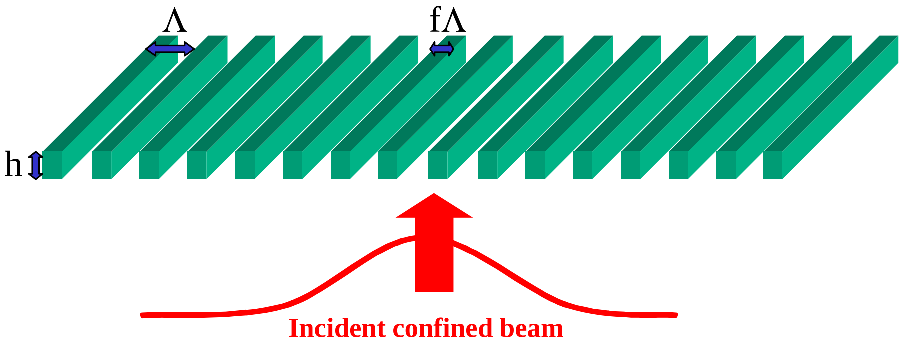

Fig. 1. Laterally confined beam impinging normally onto an infinite segmented membrane.

The segmented membrane of figure 1 has the aspect of a photonic crystal exhibiting a band gap for a wave trapped in the membrane and propagating along the latter perpendicularly to the slits. This however is not the point of view which will be adopted in what follows. It will first be assumed that a plane wave impinges onto the segmented membrane. It is known that 100% reflection can always be obtained provided the incident wavelength, the period and the incidence angle satisfy the synchronism relationship for the excitation of a guided mode of the pierced membrane. Exciting the same structure by means of a focused beam, as can be modelled by resorting to a FDTD code, reveals that the resonance is still there but the reflection has dropped down to a few percents and the reflection bandwidth has broadened.

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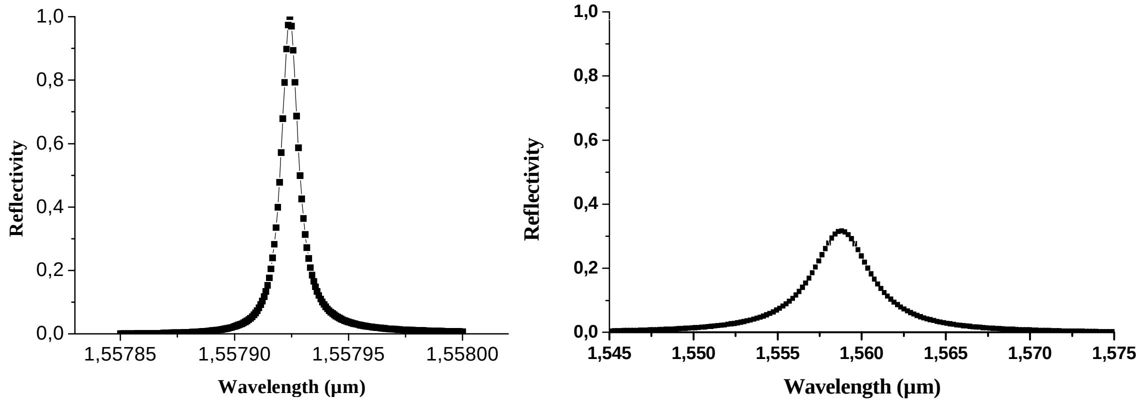


2-a) 2-b)

Fig. 2. Field reflection spectrum of a pierced silicon membrane. a) Plane waves modelling, b) Focused beam modelling.

Figure 2 illustrates the degradation of the reflection peak from a plane wave to a focused beam under normal incidence onto the same structure (h = 0.59 µm, Λ = 870 µm, f = 0.3, n_H = 3.47, n_L = 1.45 where n_H and n_L is the refraction index of the membrane and of the surrounding material respectively). It was demonstrated theoretically that this degradation is not unavoidable [1] and the evidence of a large reflection peak is demonstrated experimentally at terahertz frequencies in a companion paper [2]. The understanding of the difference between curves a) and b) gives the key for restoring high and narrow band reflection. The difference lies in the possibility for the coupled guided wave in the confined beam case to propagate away from the impact zone of the incident beam where the field which it radiates outside the membrane can not interfere with the limited incident and transmitted fields. The diffractive events in the whole process, as illustrated in figure 3, are the coupling of the incident beam to the guided mode through the first diffraction order of the periodic structure (this first order coupling is characterized by the radiation coefficient α), and, secondly, the intraguide coupling of the guided mode to itself in the contradirectional direction because of normal incidence via the second diffraction order characterized by the coupling strength κ [3].

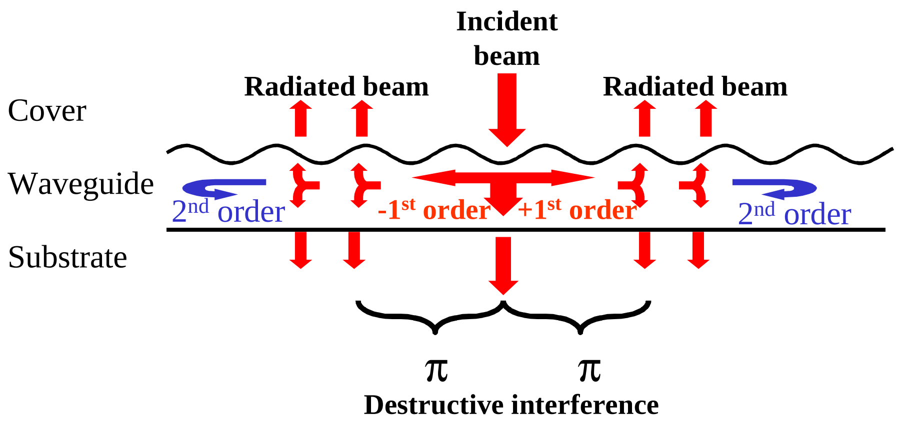


Fig. 3. Symbolic representation of the two diffraction events taking place under normal incidence: 1^{st} order in-coupling and 2^{nd} order intraguide coupling.

In the case of normal incidence the simultaneous first order coupling to the right and left propagating modes produces in the segmented waveguide a standing wave which in turns radiates into the incident and transmitted media. The lateral extend of this standing wave depends on the strength of the second order intraguide coupling coefficient.

Keeping the standing wave within the impact zone of the incident beam is the condition for a high contrast interference with the incident beam, thus for large and possibly 100% reflection. The design task is therefore twofold: first to confine the guided wave laterally by achieving a large second order coupling coefficient κ in the periodically segmented membrane, then to achieve a relatively small first order coupling coefficient α so as to increase the wavelength selectivity of the field accumulation process in the segmented waveguide. This condition can be translated into 1D photonic crystal concept and wording: resonant reflection will be achieved when the incident beam first diffracted order excites a guided mode of the segmented membrane just at the edge of its photonic conduction band. The photons coupled at the minimum of the conduction band have zero group velocity and are consequently mainly localized under the incident beam impact where from the field reemits and interferes with a definite phase relationship with the zeroth order transmitted and reflected beams. The laterally confined incident beam corresponds to an angular distribution of incidence angles of definite aperture around the normal. Those large angles correspond to non-zero group velocity, therefore to photons escaping the impact zone of the incident beam. The way to laterally trap these photons is to decrease the curvature of the conduction band, i.e. to open the gap further, which amounts to increasing the second order coupling coefficient κ [3].

3. DESIGN OF THE SEGMENTED MEMBRANE

Off-resonance, the segmented membrane represents for a normally incident TE polarized wave a thin layer of average index n_a whose spectral response exhibits maxima and minima. The reflection peak(s) which one wants to give rise to will be superimposed onto this base line. For a wave guided in the membrane, normally to the slits, the periodical segments represent a periodical multilayer exhibiting a stop band centred at the wavelength λ_{SB} which depends on the period of the stack of low and high index n_L and n_H. The first order coupling synchronism condition under normal incidence writes $K_g = \beta$ where $K_g = \frac{2\pi}{\Lambda}$ and β is the propagation constant of the coupling mode. The intra-guide coupling condition writes $2\beta = mK_g$ where for obvious reasons m = 2 which implies that the multilayer mirror must operate at its second order. This means that the multilayer period must be of the type ${}^{\lambda_{SB}}\!/_4 - {}^{3\lambda_{SB}}\!/_4$ in terms of optical thickness, i.e., ${}^{3\lambda_{SB}}\!/_4 = f\Lambda n_H = (1-f)\Lambda n_L$ which sets the fill factor f:

$$f = \left(1 + \frac{n_H}{3n_L}\right)^{-1} \qquad (1)$$

The next task is the evaluation of the period Λ. Λ is the sum of the physical width of the high and low index ridge and slit ($f\Lambda$ and $(1-f)\Lambda$ resp.). These widths are related with the wavelength limits λ_L of the reflection band of the dielectric multilayer (i.e. of the band gap of this 1D photonic crystal). These in the plane wave case can be calculated analytically. The same approach was used in the present ${}^{\lambda_{SB}}\!/_4 - {}^{3\lambda_{SB}}\!/_4$ case as that by Yeh [3] in the ${}^{\lambda_{SB}}\!/_4 - {}^{\lambda_{SB}}\!/_4$ case. Some algebraic manipulation delivers the equation giving λ_L at either side of the reflection band

$$(1+N)\cos^2\left(n_H f\Lambda \frac{4\pi}{3\lambda_L}\right) + \frac{1}{2}(1-N)\cos\left(n_H f\Lambda \frac{4\pi}{3\lambda_L}\right) - \frac{1}{2}(3+N) = 0 \qquad (2)$$

where $N = \frac{1}{2}\left(\frac{n_L}{n_H} + \frac{n_H}{n_L}\right)$.

The only meaningful solution of this quadratic equation is

$$\cos\left(n_H f\Lambda \frac{4\pi}{3\lambda_L}\right) = -\frac{(N+3)}{2(N+1)}. \qquad (3)$$

Setting $\lambda_L = \lambda_R$ as the wavelength λ_R where resonant reflection is expected to take place (for instance λ_R =1.55 µm corresponding to the minimum of the conduction band) allows the retrieval of the period Λ

$$\Lambda = \frac{3\lambda_R}{4\pi n_H f}\left\{2\pi - \mathrm{Arc}\cos\left[-\frac{N+3}{2(N+1)}\right]\right\}. \qquad (4)$$

The upper limit λ_S of the reflection band, $\lambda_L = \lambda_S$, corresponding to the maximum of the valence band, can be determined by using the second solution of (3)

$$\frac{1}{\lambda_S} = \left(\frac{3}{4\pi n_H f\Lambda}\right) \mathrm{Arc}\cos\left[-\frac{N+3}{2(N+1)}\right]. \tag{5}$$

This allows the central stop band wavelength λ_{SB} to be calculated as

$$\frac{1}{\lambda_{SB}} = \frac{1}{2}\left(\frac{1}{\lambda_R} + \frac{1}{\lambda_S}\right) \tag{6}$$

Knowing the period Λ and the fill factor f, one can calculate the off-resonance average index n_{av} therefore deduce the membrane height h which will give a base line reflection minimum at the resonant reflection wavelength λ_R, i.e.,

$$h = \frac{\lambda}{n_{av}} \tag{7}$$

with

$$n_{av} = \sqrt{f n_H^{\ 2} + (1-f) n_L^{\ 2}}. \tag{8}$$

It must be pointed out that the multilayer modelling is made under the assumption of plane waves whereas the wave trapped in the membrane « sees » the high index segments as having a lower effective index n_e because of the guided field evanescent part. The value of the fill factor can be approximately kept but the period Λ has to be corrected. The correction can be roughly made by replacing n_H in expressions (2) and (3) by n_e. This causes a correction factor of about +10% on Λ in the typical case of a silicon membrane.

3. EXAMPLE

The example chosen is a reflection filter in the THz frequency range. The desired filter wavelength is centered at 600 µm. It is a single crystal silicon structure fully fabricated by microsystem technology (n_h = 3.415 @ λ = 600 µm). The grid is suspended in air (n_l = 1).

Expression (1) provides f = 0.467. The result of expression (4) is Λ = 331 µm, the corrected value is Λ= 365 µm. From (7) and (8), the thickness is found to be h = 245µm. The position and the shape of the reflection peak is now adjusted numerically with a plane wave model. The new parameters are f = 0.465, Λ = 385 µm and h = 210 µm. The corresponding resonance peak with a plane wave code is centred at λ = 597.6 with a half maximum band with $\delta_{1/2}(\lambda)$ = 0.1 µm and exhibits 100% reflection.

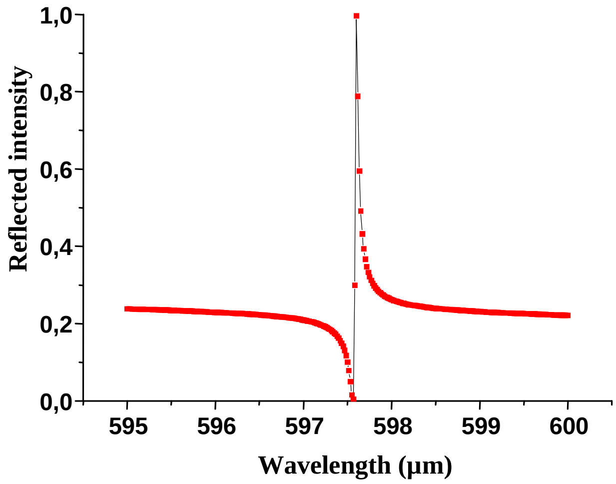


Fig. 4. Narrow spectral linewidth abnormal reflection from a segmented silicon waveguide embedded in air for a TE plane wave. The parameters are f = 0.465, Λ = 385 µm and h = 210 µm.

To check now the reflection of a confined beam, a FDTD code was used. The corresponding structure is illustrated in figure 5. The grating length is 15.75 mm, corresponding to about 41 periods. The incident TE confined beam is simulated as the fundamental mode of a 4 mm width symmetric slab waveguide of core index 1.01 and adjacent medium index 1.00 giving a mode width of 5.35 mm at 600 µm wavelength corresponding to 14 grids periods.

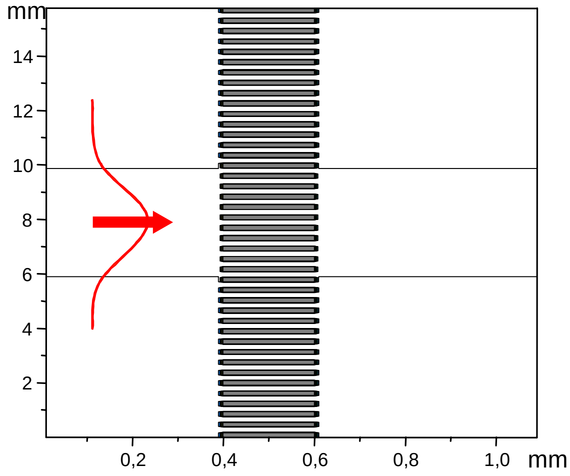
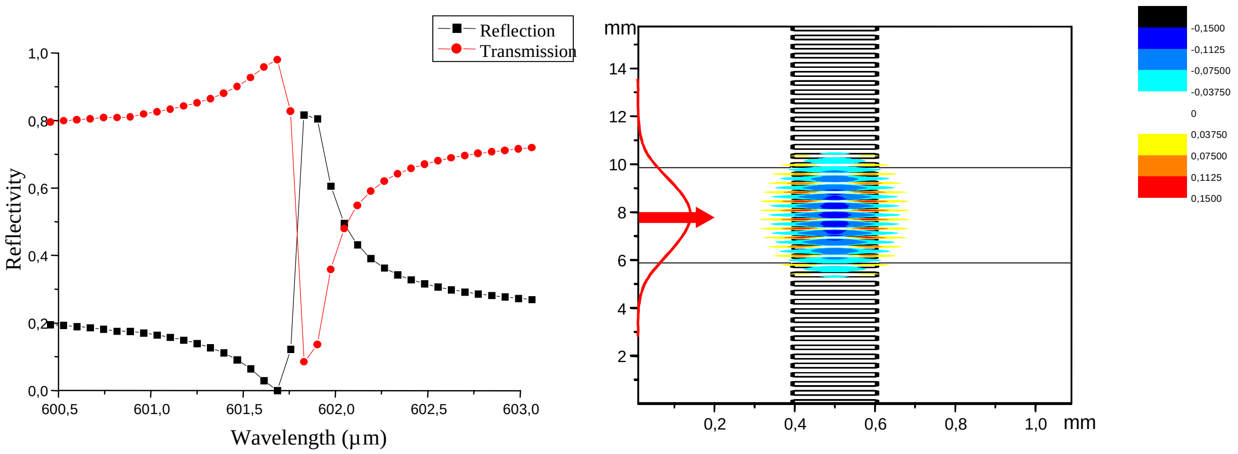


Fig. 5. Representation of the structure submitted to the FDTD code: The TE_0 modal fied is incident from the left normally to the pierced silicon membrane.

The parameters actually considered in the FDTD simulation are close to those used in the plane wave model : f = 0.473, Λ = 380 µm and h = 210 µm.

The associated spectral response is plotted in figure 6a. The spectral step is not small enough to finely describe the response shape, but the result is obtained after the reasonable calculation time of 36 hours. The reflection peak characteristics are $\delta_{1/2}(\lambda)$ = 0.29 µm and a maximum reflection larger than 80%.



a) b)

Fig. 6. a) Spectral response of the structure excited with a confined beam of about 15 periods width. b) Field distribution in the structure at resonance.

Figure 6 illustrates the field at resonance: as designed, the guided field is laterally trapped under the incident beam.

CONCLUSION

The reflection of a focused free space beam can be made narrow band, polarization selective and close to 100% by means of a high index contrast segmented grid [1] of appropriate thickness, period and fill factor. The design of such multiparameter finite beam resonant structure is usually made through a trial and error process which is very time consuming as the FDTD is most often used. The numerical search for the operation point is however made much faster by resorting to the eigen mode expansion method [5][6].

The present paper shows that the synthesis of the structure can straightforwardly be achieved analytically from a phenomenological standpoint based on a coupled wave representation. The phenomenon of a narrow band resonant reflection of a focused beam by means of a segmented membrane involving no more than 10-20 periods is obtained by laterally trapping the weakly 1st order coupled wave guided in the pierced membrane by means of a strong 2nd order intra-guide coupling. The latter is provided by a (quarter wave) / (3 quarter wave) slit/bar sequence within one grid period. Once the functional element has been approximately defined, it will then be submitted to an exact numerical method for optimisation and tolerancing purposes. The scaling rules derived from this phenomenological approach of a 1D photonic crystal have been applied for the achievement of the first experimental demonstration of the said resonant effect in the domain of terahertz frequencies in the form of a suspended silicon grid [2].

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