# Reflection of a finite light beam from a finite waveguide grating 

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#### Abstract

The method is described and the results are given of a calculation of the diffraction of a finite Gaussian light beam by a finite grating. It is shown that a waveguide grating can be used as a mirror in a plane - plane cavity. An experimental demonstration is reported of the operation of a waveguide mirror in a dye microlaser.


## 1. Introduction

The total reflection of light from the surface of a corrugated waveguide, discovered over ten years ago [1], is still being investigated intensively because of practical applications of this phenomenon in a number of optical devices [2]. The problem of reflection of finite-size light beams from infinite waveguide gratings has been studied quite thoroughly [3], but the reflection of such beams from finite waveguide gratings has not yet been discussed. A similar problem of coupling light into a waveguide by a finite grating with simultaneous excitation of oppositely directed modes was considered earlier [4] for normal incidence of a Gaussian light beam, which is of practical interest and important. We shall also limit ourselves to a near-normal incidence of a light beam on a grating. After solving the problem of reflection of light, we shall tackle the coupling of light into a waveguide. This will allow us to verify our results.

## 2. Theoretical analysis

Fig. 1 shows schematically the interaction of a light beam with a finite waveguide grating. We shall begin by considering the normal incidence of a finite light beam on an infinite waveguide grating on the assumption that we know the solution to the problem of diffraction of a plane wave incident on such a grating at an angle $\theta$. The diffraction process can then be described by the response coefficients $G^{\rho}(\rho=R, T$, $E, H$ ), where $G^{R}=R(\theta)$ and $G^{T}=T(\theta)$ are the complex amplitudes of the reflected $(R)$ and transmitted $(T)$ waves; $G^{E}=E(\theta)$ and $G^{H}=H(\theta)$ are the complex amplitudes of the electric $(E)$ and magnetic $(H)$ fields inside the waveguide. The guided modes, which are the eigensolutions of the problem, correspond to poles of the dependences of the response coefficients on the projection of the wave vector

[^0]

Figure 1. Interaction of a cylindrical light beam with a waveguide grating of finite dimensions.
of the incident wave [5]. When two guided modes are excited simultaneously, these dependences have two closely spaced poles and near these poles the response coefficients can be expanded into series:
$G^{\rho}(\beta)=\frac{b_{1}^{\rho}}{\tilde{a}_{1}+\mathrm{i} \beta}+\frac{b_{2}^{\rho}}{\tilde{a}_{2}+\mathrm{i} \beta}+c_{0}^{\rho}+c_{1}^{\rho}(\mathrm{i} \Delta \beta)+c_{2}^{\rho}(\mathrm{i} \Delta \beta)^{2}+\ldots$,
$G^{\rho}(\Delta \beta)=\frac{b_{1}^{\rho}}{a_{1}+\mathrm{i} \Delta \beta}+\frac{b_{2}^{\rho}}{a_{2}+\mathrm{i} \Delta \beta}+c_{0}^{\rho}+c_{1}^{\rho}(\mathrm{i} \Delta \beta)+c_{2}^{\rho}(\mathrm{i} \Delta \beta)^{2}+\ldots$,
where

$$
\begin{align*}
& \Delta \beta=\beta-\beta_{0} ; \beta=k n_{\mathrm{c}} \sin \theta ; \\
& \beta_{0}=k n_{\mathrm{c}} \sin \theta_{0}=-0.5 \operatorname{Im}\left(\tilde{a}_{1}+\tilde{a}_{2}\right) ; a_{1}=\tilde{a}_{1}-\mathrm{i} \beta_{0} ;  \tag{2}\\
& a_{2}=\tilde{a}_{2}-\mathrm{i} \beta_{0} ;
\end{align*}
$$

$\operatorname{Re} \tilde{a}_{1}$ and $\operatorname{Re} \tilde{a}_{2}$ are the coefficients representing the total losses experienced by the guided modes or the widths of the poles; $\operatorname{Im} \tilde{a}_{1}$ and $\operatorname{Im} \tilde{a}_{2}$ are the positions of these poles or resonances; $b_{1}^{\rho}$ and $b_{2}^{\rho}$ are the coupling coefficients of the relevant amplitudes of the waves and of the incident plane wave; $c_{0}^{\rho}$, $c_{1}^{\rho}$, and $c_{2}^{\rho}$ are complex coefficients; $\beta$ is the mismatch of the propagation constant $\beta ; \theta_{0}$ is the average 'resonant' angle; $k=2 / \lambda$ is the wave number; $n_{\mathrm{c}}$ and $n_{\mathrm{s}}$ are the refractive indices of the incident and transmitted waves.

When we deal with the interaction of a light beam with an infinite waveguide grating, the beam is usually represented by a complex amplitude $q(z)$ in the plane of the grating, satisfying the normalisation condition:

$$
\begin{equation*}
\int_{-\infty}^{\infty} q(z) q^{*}(z) \cos \theta z=1 \tag{3}
\end{equation*}
$$

We shall at first assume that the amplitude of the incident beam is equal to zero outside the region $[0, L]$, i.e. we shall consider a finite beam. Representation of the amplitude $q(z)$ in the form of a sum of plane waves

$$
\begin{equation*}
q(z)=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\infty} Q(\Delta \beta) \exp (\mathrm{i} \Delta \beta z) \beta \tag{4}
\end{equation*}
$$

where $Q(\beta)$ is the Fourier transform of $q(z)$, makes it possible to write down the response function $f^{\rho}(z)$ :

$$
\begin{equation*}
f^{\rho}(z)=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\infty} G^{\rho}(\Delta \beta) Q(\Delta \beta) \exp (\mathrm{i} \Delta \beta z) \beta \tag{5}
\end{equation*}
$$

The Fourier transforms of the functions described by the set of expressions (1) and the convolution properties of the functions yield
$f^{\rho}(z)=b_{1}^{\rho} f_{1}(z)+b_{2}^{\rho} f_{2}(z)+c_{0}^{\rho} q(z)+c_{1}^{\rho} q^{\prime}(z)+c_{2}^{\rho} q^{\prime \prime}(z)+\ldots$,
where

$$
\begin{align*}
& f_{\mathrm{i}}(z)=\left\{\begin{array}{l}
F_{L \mathrm{i}} \exp \left[-a_{1}(z-L)\right],,^{L} \leq z \\
\int_{0}^{z} \exp \left[-a_{1}\left(z-z^{\prime}\right)\right] q\left(z^{\prime}\right) z^{\prime}, \quad 0<z<L, \quad\left(\operatorname{Re} a_{1}>0\right) ; \\
0, \quad z \leq 0,
\end{array}\right.  \tag{7}\\
& F_{L \mathrm{i}}=\int_{0}^{L} \exp \left[-a_{\mathrm{i}}\left(L-z^{\prime}\right)\right] q\left(z^{\prime}\right) z^{\prime} \tag{8}
\end{align*}
$$

or

$$
\begin{align*}
& f_{1}(z)=\left\{\begin{array}{l}
0, \quad L \leq z, \\
\int_{z}^{L} \exp \left[-a_{1}\left(z-z^{\prime}\right)\right] q\left(z^{\prime}\right) z, \quad 0<z<L, \quad\left(\operatorname{Re} a_{1}<0\right) ; \\
\left.F_{01} \exp \left[-a_{1}\right]\right], \quad z \leq 0,
\end{array}\right.  \tag{9}\\
& F_{0 \mathrm{i}}=\int_{0}^{L} \exp \left[a_{\mathrm{i}} z^{\prime}\right] q\left(z^{\prime}\right) z^{\prime} . \tag{10}
\end{align*}
$$

Formulas (6)-(10) give the solution for the response functions of a finite beam incident on an infinite waveguide grating. Outside the illuminated part of this grating the equations yield two response functions, which can be regarded as the eigensolutions which exist for a corrugated waveguide (modes I and II, governed by the poles $a_{1}$ and $a_{2}$, respectively):

$$
\begin{align*}
& f_{\mathrm{I}}^{\rho}(z)=C_{1} b_{1}^{\rho} \exp \left(-a_{1} z\right)  \tag{11}\\
& f_{\mathrm{II}}^{\rho}(z)=C_{2} b_{2}^{\rho} \exp \left(-a_{2} z\right)
\end{align*}
$$

If for the normal incidence of a light beam we assume that $\operatorname{Re} a_{1}>0$ and $\operatorname{Re} a_{2}<0$, then

$$
\begin{align*}
& C_{1}=F_{L 1} \exp \left(a_{1} L\right),,^{L} \leq z, \\
& C_{2}=F_{02}, z \leq 0 . \tag{12}
\end{align*}
$$

These modes are described by the following response functions:

- the amplitude of a wave emitted into a medium with a refractive index $n_{\mathrm{c}}$ and proportional to $b_{1}^{R}$;
- the amplitude of a wave emitted into a medium with a refractive index $n_{\mathrm{s}}$ and proportional to $b_{1}^{T}$;
- the complex amplitude of the electric $(E)$ and magnetic $(H)$ fields inside the waveguide, proportional to the coupling coefficients $b_{1}^{E}$ and $b_{1}^{H}$, respectively.

The energy flux inside the waveguide can be described by

$$
\begin{equation*}
S(z)=h_{\mathrm{e}} \operatorname{Re}\left\{f^{E}(z)\left[f^{E}(z)\right]^{*}\right\}, \tag{13}
\end{equation*}
$$

where $h_{\mathrm{e}}$ is the effective thickness of the waveguide. The positive sign of $S(z)$ corresponds to the energy flow in the direction of the $z$ axis and the negative sign corresponds to the opposite direction. If a waveguide does not introduce additional dissipative losses, the energy flux in the waveguide at the edge of the unilluminated region should be equal to the total energy emitted by the whole of this region. We therefore have

$$
S(L)=h_{\mathrm{el}}\left|F_{L 1}\right|^{2} \operatorname{Re}\left[b_{1}^{E}\left(b_{1}^{H}\right)^{*}\right]=P_{1}^{R}+P_{1}^{T}
$$

for the region (labelled ' 1 ') on the right,

$$
\begin{equation*}
S(0)=h_{\mathrm{eII}}\left|F_{02}\right|^{2} \operatorname{Re}\left[b_{2}^{E}\left(b_{2}^{H}\right)^{*}\right]=-P_{2}^{R}-P_{2}^{T} \tag{14}
\end{equation*}
$$

for the region (labelled ' 2 ') on the left,
where the energy flowing into the media with the refractive indices $n_{\mathrm{c}}$ and $n_{\mathrm{s}}(\rho=R, T)$ is

$$
\begin{align*}
& P_{1}^{\rho}=N^{\rho} \frac{\left|F_{L 1}\right|^{2}\left|b_{1}^{\rho}\right|^{2}}{2\left|\operatorname{Re} a_{1}\right|} \cos \theta_{1}^{\rho} ;  \tag{16}\\
& P_{2}^{\rho}=N^{\rho} \frac{\left|F_{02}\right|^{2}\left|b_{2}^{\rho}\right|^{2}}{2\left|\operatorname{Re} a_{2}\right|} \cos \theta_{2}^{\rho} ;  \tag{17}\\
& \sin \theta_{1}^{\rho}=-\frac{\operatorname{Im} a_{1}}{k n_{\rho}} ; \sin \theta_{2}^{\rho}=-\frac{\operatorname{Im} a_{2}}{k n_{\rho}} ;  \tag{18}\\
& N^{R}=1 ; \\
& N^{T}= \begin{cases}n_{\mathrm{s}} / n_{\mathrm{c}}, & \mathrm{TE}, \\
n_{\mathrm{c}} / n_{\mathrm{s}}, & \mathrm{TM} .\end{cases} \tag{19}
\end{align*}
$$

Substitution of expressions (16) and (17) into relationships (14) and (15) gives

$$
\begin{align*}
& h_{\mathrm{eI}}=\frac{\left|b_{1}^{R}\right|^{2} \cos \theta_{1}^{R}+N^{T}\left|b_{1}^{T}\right|^{2} \cos \theta_{1}^{T}}{2\left|\operatorname{Re} a_{1}\right| \operatorname{Re}\left[b_{1}^{E}\left(b_{1}^{H}\right)^{*}\right]},  \tag{20}\\
& h_{\mathrm{eII}}=\frac{\left|b_{2}^{R}\right|^{2} \cos \theta_{2}^{R}+N^{T}\left|b_{2}^{T}\right|^{2} \cos \theta_{2}^{T}}{-2\left|\operatorname{Re} a_{2}\right| \operatorname{Re}\left[b_{2}^{E}\left(b_{2}^{H}\right)^{*}\right]} . \tag{21}
\end{align*}
$$

For normal incidence of a light beam the effective thicknesses of modes I and II are identical, because of symmetry.

The formalism described above is valid for any double resonance (normal incidence of a light beam and a resonance between the modes propagating in the opposite directions are not essential). In discussing a finite grating below, we shall consider only the case of normal incidence of light. We then have $\operatorname{Re} a_{1} \operatorname{Re} a_{2}<0$. We shall consider the specific case when $\operatorname{Re} a_{1}>0$.

Each eigenmode consists of coupled waves propagating in opposite directions. Therefore, at the boundary of a waveguide grating one eigenmode should be converted by reflection into the other mode. The sum of the fields of the incident and reflected waves should be equal to the total field corresponding to the wave escaping from the waveguide grating (boundary conditions). Consequently, the total fields $E$ and $H$ should be in phase at the right boundary of the grating and in antiphase at the left boundary.

The general solution for an infinite grating can be written in the form of a sum of the particular solution (6) and of the two mode eigensolutions described in a form similar to expression (11). If these conditions apply at the boundaries of the illuminated area $0<z<L$, it is obvious that the
solution in this area of an infinite grating should be identical with the solution for a finite waveguide grating illuminated with an identical light beam. Therefore, the solution of the problem formulated above is
$f_{\text {tot }}^{\rho}(z)=f^{\rho}(z)+C_{1} b_{1}^{\rho} \exp \left(-a_{1} z\right)+C_{2} b_{2}^{\rho}\left[-a_{2}(z-L)\right]$
subject to the boundary conditions

$$
\begin{align*}
& \frac{f_{\mathrm{tot}}^{H}(L)}{f_{\mathrm{tot}}^{E}(L)}=\frac{H_{L}}{E_{L}}=n_{L},{ }^{z}=L  \tag{23}\\
& \frac{f_{\mathrm{tot}}^{H}(0)}{f_{\mathrm{tot}}^{E}(0)}=\frac{H_{0}}{E_{0}}=-n_{0},{ }^{z}=0
\end{align*}
$$

where $E_{L}, H_{L}, n_{L}$ and $E_{0}, H_{0}, n_{0}$ are the electric and magnetic fields of the waves escaping from the area of the waveguide without the grating ( $L<z$ and $z<0$ ), and their effective refractive indices. We shall assume that the transverse field distributions are the same for a waveguide with and without a grating.

If we use expressions (22), (7), and (9), we find that the boundary conditions (23) can be rewritten as follows:

$$
\begin{align*}
& \frac{b_{1}^{H} E_{L 1}+C_{1} b_{1}^{H} \exp \left(-a_{1} L\right)+C_{2} b_{2}^{H}}{b_{1}^{E} F_{L 1}+C_{1} b_{1}^{E} \exp \left(-a_{1} L\right)+C_{2} b_{2}^{E}}=n_{L},{ }^{z}=L,  \tag{24}\\
& \frac{b_{2}^{H} F_{02}+C_{1} b_{1}^{H}+C_{2} b_{2}^{H} \exp \left(a_{2} L\right)}{b_{2}^{E} F_{02}+C_{1} b_{1}^{E}+C_{2} b_{2}^{E} \exp \left(a_{2} L\right)}=-n_{0},{ }^{z}=0 .
\end{align*}
$$

The coefficients satisfying the above system are
$C_{1}=\frac{\left(b_{2}^{H}+n_{0} b_{2}^{E}\right)\left[\left(b_{1}^{H}-n_{L} b_{1}^{E}\right) F_{L 1} \exp \left(a_{2} L\right)-\left(b_{2}^{H}-n_{L} b_{2}^{E}\right) F_{02}\right]}{\left(b_{1}^{H}+n_{0} b_{1}^{E}\right)\left(b_{2}^{H}-n_{L} b_{2}^{E}\right)-\left(b_{1}^{H}-n_{L} b_{1}^{E}\right)\left(b_{2}^{H}+n_{0} b_{2}^{E}\right) \exp \left[\left(a_{2}-a_{1}\right) L\right]}$,
$C_{2}=\frac{\left(b_{1}^{H}-n_{L} b_{1}^{E}\right)\left[\left(b_{2}^{H}+n_{0} b_{2}^{E}\right) F_{02} \exp \left(-a_{1} L\right)-\left(b_{1}^{H}+n_{0} b_{1}^{E}\right) F_{L 1}\right]}{\left(b_{1}^{H}+n_{0} b_{1}^{E}\right)\left(b_{2}^{H}-n_{L} b_{2}^{E}\right)-\left(b_{1}^{H}-n_{L} b_{1}^{E}\right)\left(b_{2}^{H}+n_{0} b_{2}^{E}\right) \exp \left[\left(a_{2}-a_{1}\right) L\right]}$.
The normalised energy fluxes in the waveguide without a grating to the right $\left(S_{L}\right)$ and left $\left(S_{0}\right)$ of its boundaries [see expression (13)]

$$
\begin{align*}
& S_{L}=S(L)=h_{\mathrm{el}} \operatorname{Re}\left\{f_{\mathrm{tot}}^{E}(L)\left[f_{\mathrm{tot}}^{H}(L)\right]^{*}\right\},  \tag{26}\\
& -S_{0}=S(0)=h_{\mathrm{ell}} \operatorname{Re}\left\{f_{\mathrm{tot}}^{E}(0)\left[f_{\mathrm{tot}}^{H}(0)\right]^{*}\right\}
\end{align*}
$$

represent essentially the efficiencies of excitation of the investigated waveguides.

The effects of reflection and transmission of those parts of the light beam which are incident on the waveguide area without the grating can be found by applying the formulas described above and approximating the functions $R_{0}(\theta)$ and $T_{0}(\theta)$ for the reflection and transmission of a plane wave in these areas of the waveguide as follows [see formula (1)]:

$$
\begin{equation*}
R_{0}(\Delta \beta)=d_{0}+d_{1}(\mathrm{i} \Delta \beta)+d_{2}(\mathrm{i} \Delta \beta)^{2}+\ldots \tag{27}
\end{equation*}
$$

where $d_{0}^{\rho}, d_{1}^{\rho}, d_{2}^{\rho}$ are the complex coefficients $(\rho=R, T)$. The amplitudes of the reflected and transmitted waves in the regions $z<0$ and $L<z$ can be found by analogy with expression (6):

$$
\begin{equation*}
f_{\mathrm{tot}}^{\rho}(z)=d_{0}^{\rho} q(z)+d_{1}^{\rho} q^{\prime}(z)+d_{2}^{\rho} q^{\prime \prime}(z)+\ldots \tag{28}
\end{equation*}
$$

The total reflection $(R)$ and transmission $(T)$ coefficients of a light beam incident on a waveguide with a corrugated finite area then become

$$
\begin{equation*}
R=\int_{-\infty}^{\infty}\left|f_{\mathrm{tot}}^{R}(z)\right|^{2} \cos \theta^{R}(z) z \tag{29}
\end{equation*}
$$

$$
\begin{equation*}
T=\int_{-\infty}^{\infty}\left|f_{\mathrm{tot}}^{T}(z)\right|^{2} N^{T} \cos \theta^{T}(z) z \tag{30}
\end{equation*}
$$

where the function $f_{\text {tot }}^{\rho}(z)$ is given by formulas (22) and (28), depending on its argument, $N^{T}$ is given by formula (19). Therefore, the energy of fluxes of the reflected and transmitted waves, and also of the modes propagating in the grating-free areas of the waveguide are given by formulas (29), (30), and (26) for a light beam of any kind incident on a waveguide structure with a finite grating.

## 3. Numerical calculation

The theory presented above was used to write a program for a personal computer, which can be used to calculate the parameters representing the process of reflection of a cylindrical Gaussian light beam from the surface of a waveguide with a corrugated area of finite dimensions. In these calculations we considered the waveguide structure shown in Fig. 1 and consisting of a guiding tantalum oxide film ( $n_{\mathrm{g}}=2.02, h=0.17 \mu \mathrm{~m}$ ) deposited on a glass substrate ( $n_{\mathrm{s}}=1.51$ ). Above the guiding film there was a 'covering' medium with a refractive index $n_{\mathrm{c}}=1.43$. The boundary between this medium and the film was corrugated with a period $\Lambda=0.33 \mu \mathrm{~m}$ (the corrugations were assumed to be rectangular and of depth $2 \sigma=10,20,30$, and 60 nm ).

Fig. 2 gives the spectral dependences of the efficiency of excitation of the guided modes, and of the reflection and transmission coefficients calculated for a Gaussian light beam incident normally on waveguide gratings with different periods $2 \sigma$. The length of the grating and the beam parameters were fixed and they were found by optimising the maximum efficiency of the mode excitation in a waveguide without a grating and for a structure with the corrugation depth $2 \sigma=20 \mathrm{~nm}$. We found that the beam axis should pass through the centre of the grating and the beam diameter $2 w=193 \mu \mathrm{~m}$ should be practically identical with the grating length $L=198 \mu \mathrm{~m}$ and match the reciprocal of the coupling coefficient of use in a corrugated waveguide [4]. A shift of the reflection (transmission) band on the wavelength scale observed for these dependences is the result of a change in the effective thickness of the guiding film. The reflection of light becomes stronger for a deeper grating, whereas a


Figure 2. Spectral dependences of the transmission (1) and reflection (2) coefficients, and of the efficiency of excitation of guided modes (3), calculated for a Gaussian light beam incident along the normal on a waveguide grating with three different grating depths.
reduction in the grating depth increases the transmission of light by the waveguide mirror, but the efficiency of excitation of the guided modes falls in both cases.

Table 1 lists the maximum reflection coefficients calculated for fixed beam parameters and the corresponding wavelengths, considered as a function of the corrugation depth. Fig. 3 shows the dependences of the reflection and transmission coefficients, and of the fraction of the incident light transformed into the guided modes, on the length of the grating mirror. These dependences reveal that there is an optimal (from the point of view of efficiency of excitation of the guided modes) grating length [4]. The attainable values of $R$ and $T$ are approximately the same ( $\sim 30 \%$ ). An increase in the grating size may raise the reflection coefficient to $95 \%$. Such a reflection coefficient can be obtained for shorter gratings with a greater corrugation depth (Table 1).

Table 1. Calculated maximal reflection coefficients and the maxima of the wavelengths of light in the excitation of a waveguide with a corrugation depth $2 \sigma$.

| Corrugation depth <br> $2 \sigma / \mathrm{nm}$ | Reflection coefficient <br> $R(\%)$ | Wavelength $\lambda / \mathrm{nm}$ |
| :--- | :--- | :--- |
| 10 | 6.43 | 590.520 |
| 20 | 25.45 | 587.419 |
| 30 | 60.88 | 584.582 |
| 60 | 92.83 | 574.189 |


$R, T, S_{0}+S_{L}$

Figure 3. Dependences of the reflection $(R)$ and transmission $(T)$ coefficients, and also of the efficiency $\left(S_{0}+S_{L}\right)$ of excitation of guided modes on the length of a grating of constant depth $2 \sigma=20 \mathrm{~nm}$. The diameter of the incident light beam is equal to the grating length, the axis of this beam passes through the middle of the grating, and the wavelength $\lambda=587.558 \mathrm{~nm}$ of the radiation corresponds to the reflection maximum of a plane wave by an infinite grating.

The case of an asymmetric position of a light beam relative to the grating centre is very interesting from the point of view of practical use. As mentioned earlier, we can then expect an increase in the intensity of one of the excited modes relative to the other. Fig. 4 gives the dependences of $R, T$, and of the mode excitation efficiency on the grating length. We can see that an increase in the grating length can increase the efficiency $(27 \%)$ of excitation of one of the modes. The reflection coefficient of the incident beam also increases, but not by very much: from $\sim 25 \%$ for the optimal grating length ( $L=198 \mu \mathrm{~m}$ ) to $\sim 40 \%$ for $L=\infty$. As the beam centre moves away from the edge of a grating, the reflection coefficient increases to $\sim 67 \%$ for $L=\infty$.


Figure 4. Dependences of the reflection $(R)$ and transmission $(T)$ coefficients, and also of the efficiencies of excitation $\left(S_{0}, S_{L}\right)$ of guided modes on the grating length (the radius of the beam and the position of its centre relative to the front edge of the grating are fixed and have optimal values; the wavelength $\lambda=587.705 \mathrm{~nm}$ of the radiation corre-sponds to the maximum efficiency of excitation of the guided modes).

Restriction of the size of the grating on a waveguide should undoubtedly influence the form of the reflected and transmitted light beams. Our theory makes it possible to estimate the changes in these beams. Fig. 5 shows the distributions of the moduli of the amplitude of these beams on the assumption that the beam profile is Gaussian and that it is incident along the normal to a waveguide grating $L=0.5 \mathrm{~mm}$ long. It follows from Fig. 5 that a beam transmitted right through a grating mirror undergoes major changes. Within the grating boundaries, the distribution of the beam amplitude now has dips and their positions coincide with the maxima of a light flux propagating in a grating waveguide. Zero value of the flux coincides with the maximum of the reflected beam amplitude. The optical energy travels along the mirror in two opposite directions: to the right and left of the point corresponding to zero value of the flux $S$.


Figure 5. Distributions of the moduli of the amplitudes of the incident (1), reflected (2), and transmitted (3) light beams, and of the normalised transverse energy flux in a corrugated waveguide (4).

## 4. Experiments

An investigation of the important (in practice) characteristics of the reflection of light at near-normal angles of incidence requires light sources capable of fine tuning or corrugated waveguides with continuously varying parameters. This
requirement arises because of the small spectral width of the observed resonant process. The conditions for a resonance can be satisfied by lasing in which a waveguide grating is used as one of the mirrors of a plane - plane cavity of a dye laser. A change in the dimensions of the grating mirror (grating length) alters its reflection coefficient and, consequently, the threshold pump energy. Therefore, the measured pump energy can be used to estimate the reflection coefficient. The intensity of a guided mode near the grating edge also carries information on the efficiency of coupling of light by the grating, considered as a function of the diameter and the axis position of the incident light beam. The width of the spectrum of the radiation generated in this way is a measure of the resonant-reflection width. Therefore, the laser method can give directly the efficiency of utilisation of a waveguide grating mirror in a plane-plane cavity.

In our experiments (Fig. 6) we used a $\mathrm{Ta}_{2} \mathrm{O}_{5}$ guiding film ( $n_{g}=2.02, h=0.17 \mu \mathrm{~m}$ ) deposited on a glass substrate ( $n_{\mathrm{s}}=1.51$ ). The upper boundary of the film was corrugated with a period $\Lambda=0.33 \mu \mathrm{~m}$. The active medium was an ethylene glycol solution of rhodamine 6 G (R6G). The concentration of this solution was $\sim 10 \mathrm{~mol} . \%$, its refractive index was $n_{\mathrm{c}}=1.43$, and it was placed between the waveguide and an aluminium mirror. This mirror and the corrugated waveguide together formed a plane - plane cavity. The thickness of the liquid active layer was $100 \mu \mathrm{~m}$ and the dye was pumped by the second harmonic ( $\lambda_{\mathrm{p}}=0.53 \mu \mathrm{~m}$ ) of an $\mathrm{Nd}^{3+}$ :YAG laser. The pump radiation was incident along the normal to the plane of the structure across the corrugated waveguide substrate. The minimum pump beam diameter in our experiments was $100 \mu \mathrm{~m}$. The diameter of the light beam with the TE polarisation and the $\lambda=590 \mathrm{~nm}$ wavelength (and a half-width less than 0.1 nm ) generated in the dye was also $\sim 100 \mu \mathrm{~m}$.


Figure 6. Experimental setup.

The lasing wavelength ( $\lambda_{\text {las }}=\Lambda n^{*}$ ) coincided precisely with the maximum of the anomalous reflection of light by the waveguide grating. The previously measured effective refractive index was $n^{*}=1.788$ and the grating period was $\Lambda=0.33 \mu \mathrm{~m}$.

Initially we used a waveguide grating which was infinite, compared with the dimensions of the reflected beam, and we observed lasing for the minimum possible pump beam diameter $(100 \mu \mathrm{~m})$. Since the calculated reflection coefficient of the waveguide grating reached $\sim 67 \%$, the generated dye laser radiation was readily detected from the output grating mirror


Figure 7. Calculated dependences of the reflection $(R)$ and transmission $(T)$ coefficients and of the efficiency of excitation $\left(S_{0}\right)$ of a guided mode on the position $z_{0}$ of the centre of the incident beam relative to the front edge of a semi-infinite grating (the beam radius is fixed and equal to the optimal value, and the wavelength $\lambda=587.705 \mathrm{~nm}$ of the generated radiation corresponds to the excitation maximum of the guided modes).
of the laser structure. We then used a semi-infinite waveguide grating and investigated the generated radiation at the output mirror of the laser and inside the waveguide adjoining the grating. The pump beam diameter was kept constant. The light intensity in the waveguide was governed by the position of the centre of the pumped spot relative to the grating edge: as the spot moved away from the edge to a distance of $\sim 400 \mu \mathrm{~m}$, the intensity of the mode inside the waveguide fell practically to zero (Fig. 7).

## 5. Conclusions

The results of our investigation were: the development of a method for calculating the parameters of a reflection of a light beam from the surface of a corrugated waveguide carrying a grating of finite length, the confirmation of the feasibility of reaching high reflection coefficients (up to $\sim 95 \%$ ) of the surface of a corrugated waveguide with a short grating ( $\sim 200 \mu \mathrm{~m}$ ) that should make it possible to use such gratings in semiconductor lasers, and - finally the construction of a dye laser with a grating mirror on a waveguide characterised by a small half-width ( 0.1 nm ) of the emission line and by an emission wavelength governed by the grating period.

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